# MATHS FOR CHEMISTRY

This is a textbook on Chemistry therefore I cannot do much in the way of teaching Maths in it, but here are some of the *mathematical* tools/concepts that you will need in order to start doing Chemistry. Please study them carefully and familiarise yourself with them *intimately*!

## 1) Equality of Treatment

- If you do the same thing to both sides of a reaction equation, then you do not alter the validity of the equation at all, e.g. if we start with the equation **"$2H_2 + O_2 \longrightarrow 2H_2O$"** and multiply the equation by say 2, then we get **"$4H_2 + 2O_2 \longrightarrow 4H_2O$"** and the equation is still valid.

- Whatever we multiply the equation by, we would not alter its validity. Therefore if we multiplied the equation above by say "X" we would get $2\mathbf{X}H_2 + \mathbf{X}O_2 \longrightarrow 2\mathbf{X}H_2O$,
  and if X = $(6 \times 10^{23})$ then we would get
  $2(6 \times 10^{23})H_2 + (6 \times 10^{23})O_2 \longrightarrow 2(6 \times 10^{23})H_2O$ and (as we shall see) this equation now reads
  **2 moles** of Hydrogen molecules + **1 mole** of Oxygen molecules = **2 moles** of Water molecules

## 2) Cross Multiplication

… and what I am going to say applies only to Multiplication and Division!

- The corollary of the above rule is that "2 =10 ÷ 5 or $2 = \frac{10}{5}$" is the same as "2 x 5 = 10"
  because what we have now done is to use the rule of equality of treatment above to multiply both sides of " 2 = 10 ÷ 5 " by "5" to get " 2 x 5 = 10 ".

- Similarly, " C = N ÷ V" is exactly the same as " C x V = N "
  and because (in Chemistry) N = Amount or Number of moles of something, and what we are saying is "Concentration = Amount ÷ Volume", and "Concentration x Volume = Amount"

## 3) Moving something from one side of an equation to the other side of the equation

- If you do something to one side of an equation, then for the equation to remain valid, you must do the same thing to the other side also – therefore if you move something from one side of an equation to the other side, then you MUST CHANGE ITS SIGN because *what you are in fact doing is adding that same thing to both sides of the equation.* For example, if you start with " 9 - 7 = 2 " and add "+7" to each side of the equation you will get " (9 -7) **+ 7** = 2 **+ 7** ". Then since the "-7" and the "+7" on the left hand side of the equation cancel each other out you will get " 9 = 2 + 7 "; and, this is exactly what you would get if in the original equation you had moved the (-7) to the other side of the equation and changed its sign. Therefore if you want to solve "x - 7 = 2 ", then just throw the "-7" onto the other side and change its sign. This gives " x = 2 + 7 ", and therefore " x = 9" . *(Actually, Maths is almost as beautiful as Chemistry!)*

## 4) The order of performing different functions

- Multiplication and Division **must be performed** before Addition and Subtraction e.g.

a) For the equation " 2 + 5 (7-2) = 27", if you first multiply the (7-2) by 5 then you would get " 2 + 35 - 10 = 27" ; but if you added the "2" and the " 5" first you would get the ***WRONG*** answer!

b) Anything in brackets can also be performed first, therefore " 2 + 5 (7-2) = 27" is the same as " 2 + 5 ( 5) = 2 + 25 = 27"

- The Khan Academy is absolutely OUTSTANDING for some of its Maths and Chemistry videos. You really **ought** to have a look at https://www.khanacademy.org/math/pre-algebra/pre-algebra-arith-prop/pre-algebra-order-of-operations/v/introduction-to-order-of-operations .

## 5) The Rule of Simple Proportions ( one of the most useful mathematical tools in Chemistry)

- **There is a very easy way of doing simple proportions – so please learn it BY HEART!**
  If, for example, you are told that 0.75 moles of a substance Z have a mass of 2.784g, and you are then asked

a) *"How many moles would have a mass of 7.326g?"*, then write it out just like this (*where "X" stands for the unknown number of moles of the substance Z*).[1]

0.750 moles of Z have a mass of 2.784g, therefore
**X** moles of Z will have a mass of 7.326g [2]

$$\text{Where} \quad \mathbf{X} = \frac{\text{Number above } \mathbf{X} \times \text{Number on the same line as } \mathbf{X}}{\text{Remaining Number}}$$

$$\text{therefore} \quad \mathbf{X} = \frac{0.750 \text{ moles} \times 7.326 \text{ grams}}{2.784 \text{ grams}}$$

$$= 1.97 \text{ moles (where the grams have cancelled each other out)}$$

b) or, in the above example, you may have been asked *"what would the mass of 1.56 moles be?"*, and then please write it out just like this

0.750 moles of Z have a mass of 2.784g, therefore
1.56 moles of Z will have a mass of **X** g

$$\text{Where} \quad \mathbf{X} = \frac{\text{Number above } \mathbf{X} \times \text{Number on the same line as } \mathbf{X}}{\text{Remaining Number}}$$

$$\text{therefore} \quad \mathbf{X} = \frac{2.784\text{g} \times 1.56 \text{ moles}}{0.750 \text{ moles}}$$

$$= 5.79\text{g} \text{ (where the moles have now cancelled each other out)}$$

- **If you learn by heart the simple rule**

  $$\textbf{In a simple proportion, } \mathbf{X} = \frac{\textbf{Number above X} \times \textbf{Number on the same line as X}}{\textbf{Remaining Number}}$$

  **then you will NEVER NEVER NEVER go wrong in calculations concerning Simple Proportions.**

## 6) The concept of something to a negative power or index

- When something is written with a negative power or index, it means that you can invert the number and change the sign e.g.

  $$\mathbf{1 \times 10^{-3} = \frac{1}{10^3} \quad or = \frac{1}{1000} \quad or = 1 \div 1000}$$

  NB $10^0 = 1$, and $10^1 = 10$, and $10^2 = 100$, and $10^3 = 1{,}000$, and $10^4 = 10{,}000$ … and so on.
  [The number in the power is the same as the number of noughts, and
  the number of digits in the answer is one more than the number in the power.]

- When something is written with a negative power or index, it means that this is equivalent to dividing by the thing with the negative power or index
  e.g. ( $X \times 10^{-3} = X \div 10^3 = X \div 1000 = \frac{X}{1{,}000}$ )

[1] When referring to an unknown number, in Mathematics we use the symbols "x" or "n" or "y" (either in capital letters or in non-capitals), but in fact you can use any symbol that you want to use so long as you make it clear what it is that you are doing.
[2] The moles and the grams are called the "units" which is short for the "units of measurement".

## 7) Standard Form or Scientific Notation

Please remember that any number can be converted into a number multiplied by 10 to the appropriate power and this latter form is called "Standard form" or "Scientific Notation". For example

| | | | | Standard form/Scientific Notation |
|---|---|---|---|---|
| 12345 | = | 1.2345 x 10,000 | = | $1.2345 \times 10^4$ |
| 1234.5 | = | 1.2345 x 1,000 | = | $1.2345 \times 10^3$ |
| 123.45 | = | 1.2345 x 100 | = | $1.2345 \times 10^2$ |
| 12.345 | = | 1.2345 x 10 | = | $1.2345 \times 10^1$ |
| 1.2345 | = | 1.2345 x 1 | = | $1.2345 \times 10^0$ (and this is a demonstration of the fact that $10^0 = 1$) |
| 0.12345 | = | 1.2345 ÷ 10 | = | $1.2345 \times 10^{-1}$ (NB NO noughts after the decimal point!) |
| 0.012345 | = | 1.2345 ÷ 100 | = | $1.2345 \times 10^{-2}$ (NB Only 1 nought after the decimal point!) |
| 0.0012345 | = | 1.2345 ÷ 1,000 | = | $1.2345 \times 10^{-3}$ (NB Only 2 noughts after the decimal point!) |
| 0.00012345 | = | 1.2345 ÷ 10,000 | = | $1.2345 \times 10^{-4}$, and so on. |

**NB For a negative power/index, the number of noughts is one less than the negative power.**

## 8) Weighted Averages

- If I have five £1 coins and two £2 coins in my hand, then I would have £9 in my hand, and in my hand I would have 7 coins, and the **arithmetically weighted average** of the coins in my hand would be £1.29p because [ (5x£1) + (2x£2) ] ÷ 7 ≈ £1.29p. ("≈" is one of the commonly accepted symbols for "roughly equal to".)

- If I had 76 £35 coins and 24 £37 coins in my hand, then the **arithmetically weighted average** of the coins would be £35.48p.[3] Please work it out on a calculator for yourself! The total number of coins is 100! (I have not chosen the numbers "37" and "35" by accident – as you will see when we start talking about the Relative Atomic Mass of Chlorine!)

- If I had 7553 £35 coins and 2447 £37 coins in my hand, the **arithmetically weighted average** coin would be £35.4894p. Please work it out on a calculator for yourself! The total number of coins is 10,000, therefore the denominator is 10,000! *(I have NOT chosen the numbers "7553" and "2447" by accident – as you will see when we start talking about the Relative Atomic Mass of Chlorine!)*

- "%" stands for the Latin words "*per centum*", and in English that means "for every 100", therefore we are dealing with exactly the same sort of calculations as the two calculations above this one! Please work it out on a calculator for yourself! (The "denominator" here is 10,000.)[4]

- OK, I hope that you have got the concept of an **arithmetically weighted average** clearly in your mind, and when we get to the "Isotopic Average Mass of an element", please therefore remember that

$$\text{Arithmetically Weighted Average} = \frac{\text{The Total Sum of all the values}}{\text{The total number of individual things involved}^{5}}$$

[3] In contrast, in a "*simple average*" no account whatsoever is taken of the number of items in each category e.g. the simple average of £35 coins and £37 coins would be a £36 coin. NB There are different sorts of weighted averages such as arithmetically weighted averages/geometrically weighted averages/etc – but you do not need to know about that at 'A' Level.

[4] You can always remember the difference between "numerator" and "denominator" by remembering that *"d stands for down below and d stands for denominator"*.

[5] And this will almost always be expressed as a "percentage"!

**9) Substitution**

If you are given the value of something in a unit that is not the unit that you require, then you can convert the value into the unit that you do require by Substitution. For example, if you are given a value for the Concentration of Calcium Hydroxide in “g per cubic decimetre” (g $dm^{-3}$) and you want it in “mol per cubic decimetre” (mol $dm^{-3}$), then you use the two equations that are relevant and then substitute. The two equations that are relevant are

$C = \frac{N}{V}$, and $N = \frac{M}{RFM}$, and now in the first equation expand C and substitute for N to obtain

$C = N \times \frac{1}{V}$, □ $C = \frac{M}{RFM} \times \frac{1}{V}$

- OK, the next one is a slightly difficult one, but you do need to know the difference between the phrases “to x decimal figures” and “to x significant figures”, so please do try to see the difference.

## SIGNIFICANT FIGURES

- **What are “significant figures”?** Significant figures (sig.figs.) infer the level of accuracy justified by the measuring instrument involved.

- The way to work out how many sig. figs. any number may have in relation to a given measuring unit, is to count the number of figures involved ***(you can ignore any zeros before the number starts, but you cannot ignore any zeros that are shown thereafter)***, and **that will be the number of significant figures** involved. In the examples below I have also shown the same numbers in scientific notation (or standard form).

| | A Number | Number of Sig. Figs. | That Number in Scientific Notation |
|---|---|---|---|
| a) | 73.476 | 5 (because there are 5 numbers) | $7.3476 \times 10^1$ |
| b) | 8.2 | 2 (there are only 2 numbers here) | $8.2 \times 10^0$ (or 8.2 x 1) |
| c) | 0.345 | 3 ( please see the note below) | $3.45 \times 10^{-1}$ |
| d) | 0.04126 | 4 (only 4 numbers are involved) | $4.126 \times 10^{-2}$ (4.126 ÷ 100) |
| e) | 0.0412600 | 6 (6 numbers are involved) | $4.12600 \times 10^{-3}$ (4.126 ÷ 1000) |

**There are SIX numbers in (e) once the numbers actually start!**

- NB You may ignore **all** the zeros *before* the start of the actual number [as in (c) and (d)], but you **MUST** take into account **ALL** the zeros that are written down once a number has started i.e. **you CANNOT ignore the two zeros at the end of the number in (e).**

- Students sometimes get confused when they see a number such as 0.100 mol, or 0.100 g, or 0.100 m, etc – because, in Maths, this number can be written as 0.1 mol or 0.1 g, or 0.1 m, etc. However, in the Physical Sciences you cannot, **YOU CANNOT** and you **MUST NOT** write 0.100 mol as 0.1 mol because the way in which the number is written IMPLIES a level of accuracy of 1 part in 100 i.e. it has an accuracy of THREE SIGNIFICANT FIGURES! 0.100 mol has **THREE significant figures**.

- In the Physical Sciences $1.00 \times 10^{-1}$ mol **does not equal** $1.0 \times 10^{-1}$ mol, nor does it equal $1 \times 10^{-1}$ mol! **$1.00 \times 10^{-1}$ mol MUST be written as “$1.00 \times 10^{-1}$ mol”.**]

- OK, so why do you need to know about significant figures? You need to know about significant figures because the calculator that you use is very clever and at the same time it is also *very stupid*! If you divide "4" by "3", the answer that your calculator will give you is 1.33333333333, and if (e.g. when you are doing the calculations for a Titration) you put down 1.33333333333 mol as your answer, then you would be telling the examiner that the accuracy of the instrument that you used in your experiment was 1 in 100,000,000,000 – **and this would clearly be absolute tosh**! You would then naturally ask yourself "Yes, but to how many decimal places should I give my answer?", and the answer is that the number of decimal places is determined by the number of SIGNIFICANT FIGURES that you must use, i.e. it is determined by
  a) the LEAST accurate piece of equipment that you use in an experiment, or
  b) the LEAST number of significant figures given in the question that you are asked e.g. if the question asks "What is the percentage of Oxygen in Water?", then if your Periodic Table gives the RAM for Hydrogen to 2 sig. figs. (viz. 1.0)[6] and that for Oxygen to 3 sig. figs. (viz. 16.0), then you must give your answer to only **TWO** significant figures – and *the answer therefore is 89% (the answers "88.9%" or 88.89% or 88.889% would all give a spurious level of accuracy that is **NOT** justified!).*

- In my time I have had to mark Practicals, so I know a tiny bit about how Practicals exams are marked. In your *written exams*, the examiner may or may not have to deduct marks for an answer that is numerically correct but which does not contain the appropriate number of significant figures, but **in your Assessed Practicals, the Marking Schemes say that the assessor MUST give you a mark for using the appropriate number of significant figures in your answer**, and the assessor CANNOT give you that extra mark if you have not used the appropriate number of significant figures. [One mark out of 30 marks is only 3% (i.e. the difference between getting 97% and getting 100%), so it is no big deal especially since your Assessed Practicals represent only 20% of your overall AS Level mark – so if you find it too difficult to understand, then do not lose any sleep over Significant Figures! If however, you are keen to get every last mark available to get a place at one of the Russell group of Universities, then it is worth learning the very simple rules that govern the use of Significant Figures.]

## LOGARITHMS (or LOGS) and the concept of "p"

*The credit for the development of "logs" is given to John Napier, but in reality many great mathematicians (notably Leonard Euler) contributed to the development of 'logs'.*

**What is a "log"?**

- In the olden days, **before calculating machines were in common use**, it was very difficult to multiply and divide numbers that were large. For example, please could you try multiplying 14,725 by 12,365 by hand (i.e. without using your calculator)*! Now can you see the problem that they had in the olden days?*

- However, in the late 1500s/early 1600s at least two mathematicians (a Swiss gentleman who did not publish his thesis, and a man called John Napier), used the algorithm (I will repeat that, used the "ALGORITHM")[7]

$$[\ A^x \quad \textit{multiplied by} \quad A^y\ ] \quad = \quad A^{(x+y)}$$

  to formulate a way of making multiplication and division easier, because instead of multiplying $A^x$ and $A^y$ together, all that needs to be done now is to ADD together x and y and then look up an antilog table to find out the value of $A^{(x+y)}$. *[NB $A^x$ divided by $A^y$ would give $A^{(x-y)}$, therefore complicated division and multiplication became just as easy as each other!]*

- Napier then laboriously worked out BY HAND the logarithms and the antilogarithms (or "logs" and "antilogs") for masses and masses of numbers and then published them for everybody in the world to use.[8] **The "logs" that you use in your modern electronic calculator are a direct result of something invented by John Napier FIVE HUNDRED years ago!!!!!**

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[6] The Latin abbreviation "viz." stands for "videre licet" which today in English stands for "namely" or "in other words".
[7] From about 800-1,200 AD when we in Europe were dressing up in suits of armour and killing each other, the Muslims in the Near/ Middle were almost the sole repository of all scientific knowledge. Any word in science that starts with the letters "*al*" (i.e. the Arabic definite article) is therefore almost invariably derived from a word of Arabic origin. In that era, the translators (who could translate Greek and Latin texts into Arabic) were treated by the Muslims as we treat footballers today. *They became some of the wealthiest professionals in the whole world!*
[8] Therefore please remember that the good things that you do during your lifetime *can* live on forever!

- Clearly, any number can be chosen as the base for which a log is the power - but in the end it was found that calculating logs to the base of 10 gave the most CONVENIENT logs[9] for multiplication and division, but calculating logs to the base of "e" was the most important for **pure mathematics** because

$$\frac{d}{dx}(e^x) = e^x$$

*[NB If you have not done any Calculus, then please do not worry about the above equation for the moment.]*

- SO THAT IS IT! **Logs are nothing more than the power of 10 (or of e, or of <u>any</u> number) which gives a particular number e.g.**
  **$10^{2.0}$ = 100** therefore 2.0 is the *log-to-the-base-10* of 100, or $\log_{10} 100 = 2.0$.
  The "2" in front of the decimal point in "2.0" is called the "characteristic" (the bit that follows from the decimal point is called the "mantissa"), and it tells you that there are three numbers in front of the (unshown) decimal point in "100", and, the "5" in "5.0428" below tells you that there are six numbers in front of the decimal point in "110,357.029".
  **$10^{5.0428}$ = 110,357.029** and 5.0428 is the *log-to-the-base-10* of 110,357.029, or
  $\log_{10}$ 110,357.029 = 5.0428. *[Please check this for yourself on your calculator!]*

- OK, so how did logs help make multiplication and division easier? Well once you have the logs of numbers, it is easy to add up the logs of the two numbers (or subtract them if you are dividing) and then in the appropriate antilog table look up the antilog of the resulting number, and then you will have your answer! Modern computers did not come into being until the 1980s. I am now very near 80 years of age – and, **until the 1980s, everybody had to multiply big numbers by using logs!**

- For example please use your calculator to find the logs of the two big numbers in the first bullet point in this Appendix. You should get 4.16806 and 4.09219 – and these are the numbers that you would have got *(without doing any calculations at all)* just by looking up Napier's log tables. If you then added 4.09219 and 4.16806, you would get 8.26025, and if you then looked up Napier's antilog table for that number, you would get the answer $1.8207 \times 10^8$. Could you now multiply the two big numbers on your calculator, and you will see that you get almost exactly the same answer (or 182,074,625 to be exact) that you would have got just by looking up Napier's tables 500 years ago. *At a time when there were no calculators, Logs gave almost exactly the right answer.*

- **Can you now see how Napier absolutely transformed the execution of multiplication and division, by the invention of his "logs"![10] Without them, it would have taken ages to multiply and divide big numbers, but with them, multiplication and division became child's play – and this was CRUCIALLY important to navigators/engineers/mathematicians/gunnery officers/etc (and also to all of us who did 'A' Level Maths before electronic calculators became popular in the 1980s).**

- Let me do for you a division calculation using logs e.g. 14,725 ÷ 12,365
  Step 1 $\log_{10}$ 14,725 = 4.16806
  Step 2 $\log_{10}$ 12,365 = 4.09219
  Step 3 4.16806 - 4.09219 = 0.07587 [11] (see footnote 8)
  Step 4 $10^{0.07587}$ = 1.1909
  and if you now check this answer with the answer given by your calculator, you will see that the answer is almost exactly the same as the one above.

- ***The remainder of this note contains some modestly complicated Maths.*** **You DO need to know about "logs" for your second year of 'A' Level Chemistry when you start learning how to calculate the pH of acids and alkalis (and if you are going on to study Medicine, then you will need to know about logs for some of the subjects in Medicine – especially when you are doing Henderson-Hasselbalch equations). Please note the difference between**
  **"log (x)" which is the accepted notation for "$\log_{10}$ (x)", and "ln (x)" for "$\log_e$ (x)".**

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[9] Napier actually choose $(1\text{-}10^{-7})$ as his base (but I am not going to go into the reason for this choice), and another mathematician called Henry Briggs subsequently (with Napier's consent) changed the base to "10" – the base of modern logs.

[10] And my goodness, just think how much easier it made multiplication and division involving three or more numbers!

[11] The zero in front of the decimal point tells you that there is only one number in front of the decimal point in the answer.

Rules of logarithms

$\log(1) = 0$ $\quad$ $\ln(1) = 0$

$\log(10) = 1$ $\quad$ $\ln(e) = 1$

$\log(100) = 2$ $\quad$ $\ln(e^x) = x$

$\text{Log}(10^x) = x$

$\log A^x = x\log A$ $\quad$ $\ln A^x = x\ln A$

$$\log\left(\frac{A^x}{B^x}\right) = \log\left(\frac{A}{B}\right)^x = x\log\left(\frac{A}{B}\right)$$

$\log(AB) = \log A + \log B$ $\quad$ $\log\left(\frac{A}{B}\right) = \log A - \log B$

**What is "p" (and this is purely and simply a matter of definition)?**

| | | | | |
|---|---|---|---|---|
| | **p** | **=** | **$-\log_{10}$ (of whatever)** | |
| therefore | $pK_a$ | = | $-\log_{10}$ of $K_a$ | |
| and | pH | = | $-\log_{10}$ of $[H^+]$ | and this **gives you** the pH of an acid |
| and | pOH | = | $-\log_{10}$ of $[OH^-]$ | and pH of a base = 14.0 – pOH. |

**NB** **Strictly speaking, the last few equations have been written incorrectly.** An "=" sign is a notation from Mathematics, and the "=" signs are here used incorrectly. What should have been written is that "the value of the item on the right hand side (RHS) of the equations gives the desired value for the items on the LHS of the equations". Nevertheless, please note that this is how the equations are written in Chemistry, and that Chemists tend to be not particularly robust Mathematicians, although Physicists are often better Mathematicians than Mathematicians are.[12]

- Therefore if e.g. pKa = 4.76 (the pKa for Ethanoic Acid)
  then $-\log_{10}(K_a)$ = 4.76
  and $\log_{10}(K_a)$ = - 4.76
  then, if you remember that **$\log_{10}$ 100 = 2.0** and therefore that **$10^{2.0}$ = 100** then you can do the next bit on your calculator and get

  $$K_a = 10^{-4.76} \approx 1.74 \times 10^{-5}$$

- | | | | | |
  |---|---|---|---|---|
  | Moreover, since | $K_w$ | = | $[H^+] . [OH^-]$ | and this is something that you will learn. |
  | and since | log of (X . Y) | = | log of (X times Y) | = log X + log Y |
  | then | $\log_{10} K_w$ | = | $\log_{10}[H^+]$ + | $\log_{10}[OH^-]$ |
  | therefore | $-\log_{10} K_w$ | = | $-\log_{10}[H^+]$ - | { $\log_{10}[OH^-]$ } |
  | or | $-\log_{10} K_w$ | = | $-\log_{10}[H^+]$ + | { $-\log_{10}[OH^-]$ } |
  | therefore | **$pK_w$** | **=** | **pH** **+** | **pOH <– THIS IS VERY IMPORTANT** |

- When talking about logs, could you please note that since $\log_{10}10 = 1.0$ and $\log_{10}100 = 2.0$ and $\log_{10} 1000 = 3.0$, then there is a TENFOLD increase between one log numeral and the next one. For instance, when you hear that an earthquake was of magnitude 8 on the Richter (log) scale and that a different one was of magnitude 9, then this means that the second earthquake **was TEN TIMES more powerful the first one**! The same holds good for pH values which measure the concentration of $H^+$ ions in a given solution at a given temperature. To say that acid X has a pH of 1 whereas acid Y has a pH of 2 means that acid X has **TEN TIMES** as many $H^+$ ions in a given volume compared to acid Y. **NB The smaller the pH of an acid, the LARGER is its concentration of $H^+$ ions.**

[12] Much of Physics was developed via Mathematics – and certainly everything that we know about the atom comes from mathematical models developed by Physicists such as Bohr/Heisenberg/Schrodinger/Pauli/and (not forgetting one of the greatest British physicists ever) Paul Dirac. *(I never met Einstein, but I did meet the Nobel Laureate Lawrence Bragg in 1955 when I was just 15 years old! A lady in my home town, Gerrards Cross, was his secretary.)*

- **I do hope that this next statement is going to rock you back on your heels!** The difference between an aqueous solution with a pH of zero (a very acidic substance) and one with a pH of 14 (a very alkaline substance) is that the acid has **100,000,000,000,000** times as many $H^+$ ions per $dm^3$ in it as has the alkali.

- **If you are going to study Medicine at University, then it would be very helpful to you if you were to understand "logs". I am sorry to say that there are an awful lot of doctors (and even some Lecturers in Medical Schools) who do not understand logs, and I find this frightening since all doctors have the power of life and death in their hands!**

## There is no such thing as a negative log number

- A "log" is a power or an index. If you are using "10" as your base[13], then
  $\log_{10}$ of 1,000,000 = 6 or $10^6$ = 1,000,000 i.e. 10 to the power of 6 = 1,000,000
  $\log_{10}$ of 100,000 = 5 or $10^5$ = 100,000 i.e. 10 to the power of 5 = 100,000
  $\log_{10}$ of 10,000 = 4 or $10^4$ = 10,000 i.e. 10 to the power of 4 = 10,000
  $\log_{10}$ of 1,000 = 3 or $10^3$ = 1,000 i.e. 10 to the power of 3 = 1,000
  $\log_{10}$ of 100 = 2 or $10^2$ = 100 i.e. 10 to the power of 2 = 100
  $\log_{10}$ of 10 = 1 or $10^1$ = 10 i.e. 10 to the power of 1 = 10
  $\log_{10}$ of 1 = 0 or $10^0$ = 1 i.e. 10 to the power of 0 = 1
  and if you check this on your calculator you will see that this is so.

- Now what happens if you try to take the log of a negative number? Well, try to take the log of "-1" on your calculator and you will see that it says "ERROR MESSAGE", and the reason that you cannot do this is that **10 to the power of anything (even a negative number) always gives a positive number. You cannot raise 10 to the power of something and get a negative number. It is a mathematical impossibility.**

- OK, so what happens if we raise 10 to the power of a negative number? Well, let us do it and see what happens.
  $10^{-1}$ = 0.1 – 1 = $\log_{10}$ of 0.1
  $10^{-2}$ = 0.01 – 2 = $\log_{10}$ of 0.01
  $10^{-3}$ = 0.001 – 3 = $\log_{10}$ of 0.001
  $10^{-4}$ = 0.000,1 – 4 = $\log_{10}$ of 0.000,1
  $10^{-5}$ = 0.000,01 – 5 = $\log_{10}$ of 0.000,01
  $10^{-6}$ = 0.000,001 – 6 = $\log_{10}$ of 0.000,001

  As you can see, as we increase the size of the negative power/index, all that happens is that we get a smaller and smaller positive number – **and this number will keep getting closer and closer to "zero" but it will NEVER get to zero, and it will therefore never go below zero and become a negative number!**

- **You cannot raise 10 to the power of something and get a negative number. It is a mathematical impossibility.** However, you can get a negative pH value of an acid (which indicates that the acid is a **very** concentrated strong acid) – but I am not going to talk about that here.

NB I know nothing at all about Biology, but if you are doing Biology, then I have taken the following extract from Prof Stephen Lower's Virtual Chemistry book (Simon Fraser University, British Columbia, Canada), and you can see how Chemistry interacts with Biology. When we get to Buffer solutions and Henderson-Hasselbalch equations, please do pay **very** careful attention to these topics because if you go on and become a medic, then that will help you to avoid unintentionally killing your patients.

- **You cannot raise 10 to the power of something and get a negative number. It is a mathematical impossibility.** However, you can get a negative pH value of an acid (which indicates that the acid is a very very concentrated strong acid) – and I am not going to talk about that here, but I will tell you about it in the Upper Sixth if you decide to do A2 Chemistry.

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[13] ... but you can use any number as your base!

There are some excellent Maths videos on *youtube* that you may want to watch. I am told by colleagues that a gentleman called Eddie Woo has put one on the web on Logs – but I cannot vouch for it because I myself have not watched it.

OK, that is enough Maths. Let us start doing Chemistry.